Advanced Artificial Intelligence.
Assignment 3 – Utility and Decision Networks.

Dmitriy Bespalov

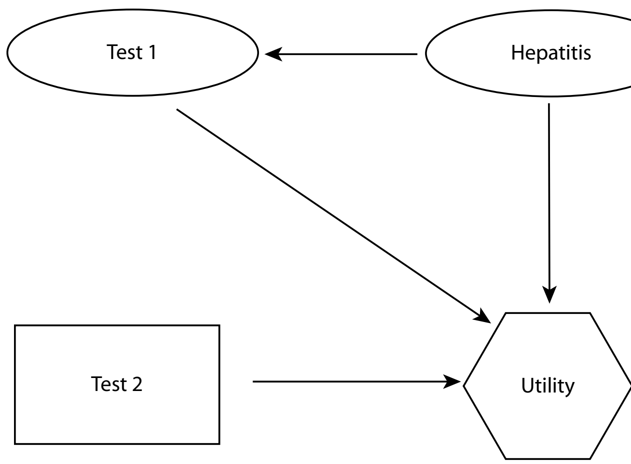

Figure 1: Decision Network.

Hepatitis	Test1	
	Positive	Negative
Present	0.8	0.2
Absent	0.2	0.8

Hepatitis	
Present	Absent
0.033	0.967

Test2	Test1	Hepatitis	Utility
Take	Positive	Present	60
Take	Positive	Absent	60
Take	Negative	Present	60
Take	Negative	Absent	60
Dot't Take	Positive	Present	100
Dot't Take	Positive	Absent	20
Dot't Take	Negative	Present	0
Dot't Take	Negative	Absent	100

Let H denote Hepatitis, $T1$ – Test1, $T2$ – Test2.

a)

$$\begin{aligned} EU(\text{conduct Test2}) = \quad & P(T1,H)\cdot U(T2,T1,H) + P(T1,\neg H)\cdot U(T2,T1,\neg H) + \\ & P(\neg T1,H)\cdot U(T2,\neg T1,H) + P(\neg T1,\neg H)\cdot U(T2,\neg T1,\neg H) \end{aligned}$$

Using formula $P(A,B) = P(B)\cdot P(A|B)$ we get:

$$EU(\text{conduct Test2}) = (0.8\cdot 0.033 + 0.2\cdot 0.967 + 0.2\cdot 0.033 + 0.8\cdot 0.967)\cdot 60 = 60$$

$$\begin{aligned} EU(\text{not conduct Test2}) = \quad & P(T1,H)\cdot U(\neg T2,T1,H) + P(T1,\neg H)\cdot U(\neg T2,T1,\neg H) + \\ & P(\neg T1,H)\cdot U(\neg T2,\neg T1,H) + P(\neg T1,\neg H)\cdot U(\neg T2,\neg T1,\neg H) \end{aligned}$$

$$EU(\text{not conduct Test2}) = 0.8\cdot 0.033\cdot 100 + 0.2\cdot 0.967\cdot 20 + 0.2\cdot 0.033\cdot 0 + 0.8\cdot 0.967\cdot 100 = 83.868$$

Expected utility for not conducting second test is greater, so it shouldn't be conducted if no info is provided.

b)

$$EU(\text{conduct Test2}) = P(H|T1)\cdot U(T2,T1,H) + P(\neg H|T1)\cdot U(T2,T1,\neg H)$$

$$P(H =< T,F > |T1) = \alpha\cdot P(H =< T,F >)\cdot P(T1|H =< T,F >) = \alpha\cdot < .0264, .1934 >=< .12, .88 >$$

$$EU(\text{conduct Test2}) = .12\cdot 60 + 0.88\cdot 60 = 60$$

$$EU(\text{not conduct Test2}) = .12\cdot 100 + 0.88\cdot 20 = 29.6$$

c)

For false-positive:

$$EU(conductTest2) = U(T2, T1, \neg H) = 60$$

$$EU(notconductTest2) = U(\neg T2, T1, \neg H) = 20$$

For false-negative:

$$EU(conductTest2) = U(T2, \neg T1, H) = 60$$

$$EU(notconductTest2) = U(\neg T2, \neg T1, H) = 0$$

False-positive influences more than false-negative on the decision that Test2 is not conducted.

d)

Assume that $P(T1|H) = P(\neg T1|\neg H)$ and $P(\neg T1|H) = P(T1|\neg H)$. That values would determine the accuracy of the test. Therefore we obrain, given that no more information is provided:

$$EU(T2) = (P(T1|H) \cdot 0.033 + P(\neg T1|H) \cdot 0.967 + P(\neg T1|H) \cdot 0.033 + P(T1|H) \cdot 0.967) \cdot 60 = 60$$

$$EU(\neg T2) = P(T1|H) \cdot 0.033 \cdot 100 + P(\neg T1|H) \cdot 0.967 \cdot 20 + P(\neg T1|H) \cdot 0.033 \cdot 0 + P(T1|H) \cdot 0.967 \cdot 100$$

If we want second test to be worth taking, we get:

$$EU(T2) > EU(\neg T2) \Leftrightarrow 60 > 3.3 \cdot P(T1|H) + 19.34 - 19.34 \cdot P(T1|H) + 96.7 \cdot P(T1|H) \Leftrightarrow$$

$$\Leftrightarrow 80.66 \cdot P(T1|H) < 40.66 \Leftrightarrow P(T1|H) < 0.504$$

So the accuracy of the first test should be $P(T1|H) < 0.504$.

Now, if we set accuracy $P(T1|H) > 0.504$ then EU will be higher for not conducting the second test unless we are given that the first test returned positive.

Using formula from **b)**, we get

$$EU(T2) = P(H|T1) \cdot 60 + P(\neg H|T1) \cdot 60 = 60$$

$$EU(\neg T2) = P(H|T1) \cdot 100 + P(\neg H|T1) \cdot 20 = 100 \cdot P(H|T1) + 20 - 20 \cdot P(H|T1)$$

$$EU(T2) < EU(\neg T2) \Leftrightarrow 60 < 80 \cdot P(H|T1) + 20 \Leftrightarrow 80 \cdot P(H|T1) > 40 \Leftrightarrow P(H|T1) > 0.5$$

Also, $P(T1|H) = \alpha \cdot < \frac{P(H|T1)}{P(H)}, \frac{P(\neg H|T1)}{P(\neg H)} >= \alpha \cdot < 15.15, 0.517 >=< 0.967, 0.033 >$

So, $P(T1|H) > 0.967$ in order for the not taking the second test will always be dominant.