Backward Mapping Methodology
for Design Synthesis

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BACKWARD MAPPING METHODOLOGY FOR DESIGN SYNTHESIS

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ABSTRACT
Design efficiency and robustness at early stage of parametric engineering design play a critical role in reducing cycle time and improving product quality in the overall product development process. Usually, the “forward mapping” approach is used to find designs, where the desirable performances are satisfied through large iterations of analysis and evaluation from design space to performance space. However, these approaches are time-consuming and involve blind search if the engineering system simulation models and/or initial conditions are not appropriately selected. On the other hand, common “reverse engineering” methods use domain-specific assumptions and are not effective in generic scenarios where the presumed conditions are violated. In this paper, a Backward Mapping Methodology for Design Synthesis (BMDS) is presented that can help conduct design synthesis rapidly and robustly at early stage of parametric engineering design. BMDS is a surrogate model-based approach that combines the strengths of metamodeling and statistics. It can help designers explicitly identify the robust design solutions that satisfy the designer-specified performance requirements through a “backward mapping” from the performance space directly to the design space. Preliminary case studies show its effectiveness and potential to be used as a generic early stage parametric design synthesis methodology in the future.

KEYWORDS
Design Synthesis, Backward Mapping, Metamodelling, Statistics, Early Stage of Parametric Engineering Design

INTRODUCTION
Parametric engineering design happens between a functional domain, which consists of a set of performance variables and their ranges (or performance space), and a physical domain, which is defined by a set of design parameters and the associated intervals (or design space). The performance requirements (design objectives) are defined in the functional domain and parametric engineering design is the process to generate feasible solution(s) in the physical domain to accomplish these objectives [SUH90]. With increased demands of frequent product model revision from
the competitive global market, most industries are using the carry-over designs in order to expedite the time to market and reduce the cost. As a result, design, for these industries, is synonymous with the search for a new set of design parameter values that satisfy the new requirements.

Parametric design at early stage, or preliminary design, plays an important role in any product design. Research has shown that design freedom decreases while cost increases as the design proceeds from the early design stage to the final design stage. It is at the early design stage that engineers have the most freedom to make design changes, explore design alternatives and choose the most appropriate design(s). An effective early design decision provides a sound foundation for the subsequent detailed design and all the other engineering decisions made later. At early stage, a large design space needs to be explored to get feasible design candidates that satisfy the given performance requirements for the later detailed design stages. Design analysis is commonly used to aid the search for the acceptable design parameters. Usually, it starts with an initial set of design parameters based on the prior design knowledge. Either physical experiments or computer simulations are performed to validate the design. If the targeted performance requirements are met, the design is accepted. If not, revisions to the design are made and new tests or simulations are performed until a desired solution is reached. Design optimization techniques are often used to optimize the process.

However, the emphasis at early stages should be on the derivation of a wealth of robust design alternatives as opposed to optimized isolated solutions. The computer-based simulation tools that can represent the system characteristics and mimic the system behavior are currently used overwhelmingly to generate, evaluate and validate designs. Since they are physics-based models, their use often requires solving a set of nonlinear partial differential equations. For a complex design problem these equations could become complicated and the computing cost is unbearable. The search for a feasible design that satisfies the given performance requirements usually involves numerous iterations among several simulation tools and an optimizer in current engineering practice. It is extremely time-consuming and limits engineers' ability to evaluate more design alternatives in a given time frame. In this scenario, an abstract model that can mimic engineering system behavior at acceptable sacrifice of accuracy/details is more useful. The most popular approach is to use a surrogate model to replace the costly model in such a situation. Usually, the surrogate model is not as detailed nor accurate as the most detailed simulation model, but much more efficient to be used. Another benefit of using surrogate model is that: it has flexible and generic representation format, which can facilitate the communications in multiple-stage, multiple-designer environments. Many metamodelling methods have been developed to build surrogate models as alternatives to the

physic-based mechanistic models in order to improve design efficiency at early stage.

The objective of rapid alternate robust solution rendering for early stage design may be attained by either reducing simulation time and/or by decreasing model iteration time. However, rapid solution rendering from the available (surrogate) models has not been addressed to date. Efficient, robust and generic methods need to be developed in order to improve design quality at the early stage of parametric engineering design.

In this paper, a Backward Mapping Methodology for Design Synthesis (BMDS) is introduced and described. It is a surrogate model-based approach to rapidly synthesize robust design alternatives at early stage of parametric engineering design. BMDS can help designers explicitly identify the most robust design regions that satisfy the designer-specified performance requirements through a direct “backward mapping” procedure from performance space to design space. In this framework, the domain knowledge of an engineering system is represented in a more comprehensive and applicable form and then design synthesis is conducted based on this representation. Due to the generic form used, BMDS is capable of cross-domain utilization and the efficiency of the design process is improved as the result of using the fast substitute. Explicit consideration for design robustness is another important feature of BMDS. Design confidence is explicitly addressed and integrated with desirable performance and its range in the performance requirements specified by the designer. Unlike other methods, set-to-set mapping is used and regions instead of single points are basic design units in this framework. The combination of metamodelling and statistics in the framework provides a feasible way to handle modeled and unmodeled uncertainty. Compared to the “forward mapping” approaches using try-and-error iterations, BMDS avoids ineffective searches and provide the designer with explicit sense of design confidence. Compared to those in “reverse engineering” [YAMA90], it is a more generic methodology that can be applied to many engineering domains. Its application in vehicle bumper system design shows its unique strengths in dealing with complex, highly nonlinear engineering system design problem at early stage of parametric engineering design.

In the following sections, the existing approaches are reviewed. Then BMDS and its application are described in details. Finally, conclusions and future work are summarized.

REVIEW OF EXISTING METHODS AND THEIR LIMITATIONS

There has been a lot of research effort in studying the complex nature of the design problems. Different design theories and methodologies have been developed to address the design efficiency issue. Among them are Metamodelling and surrogate model-based design approaches. Metamodelling has
gained more attention recently both in academia and industry applications for they can be used to build surrogate models that can serve as alternatives to the physics-based mechanistic models. Using the surrogate models built with metamodelling techniques can speed up performance prediction and therefore improve the design efficiency.

**Metamodeling**

Metamodeling, as indicated by its name, is a technique to build models of models. The basic approach is to construct approximations of the mechanistic models, which will capture the most critical relationships between a set of design parameters: x, and a set of performance parameters: y with some error [SIMPS97]. The existing metamodeling technologies include traditional statistics-based approach, kriging, machine learning-based approach and hybrid approach.

The traditional statistics-based metamodeling methods largely rely on Design of Experiment (DoE) to collect data sets from physical experiments and then apply regression analysis to fit a response surface [MONT97]. A response surface represents a relationship between a set of independent input variables x and an output variable y:

\[ y = f(x) + \varepsilon \]  

Since the true response surface function \( f(x) \) is usually unknown, a response surface \( g(x) \) is created to approximate \( f(x) \). Different approximation functions can be used [BOX87]. These kinds of surrogate models are normally used for optimization within Response Surface Methodology [MONT75] [MYERS89] [MYERS95].

Lately, computer simulations have been used to replace the physical experiments for the data collection. Since the computer simulations are deterministic and thus not subject to random error, the randomness assumption for the least square regression does not hold anymore [SIMPS97]. As an alternative, researchers are looking into a new modeling approach called “Kriging”. Instead of using equation (1), the kriging approach models a response as a combination of a polynomial model and an error model:

\[ y(x) = f(x) + Z(x) \]  

where \( y(x) \) is the unknown function of the interest, \( f(x) \) is a known polynomial function of x, and \( Z(x) \) is an error model representing the residuals. The key element of the kriging approach is to find \( Z(x) \) by interpolating the sampled data points. Different correlation functions can be used for the interpolation process. The success of this approach depends on the appropriate choice of the correlation functions, which currently rely heavily on users’ experience and in most of the cases, it is still a “guessing” process [SACKS89] [WELCH90] [WELCH92] [KOEHL96].

Machine learning-based approaches use different type of metamodeling techniques. They use training data sets to learn underlying relationships between the space of x and y, and build surrogate models thereon. These surrogate models may take many forms, such as neural networks and decision trees [MONOS96] [LU90]. They have been used in many engineering applications. In some cases, they can provide more comprehensive and accurate solutions than the statistic methods. However, insufficient training data sets, inappropriate model validation methods or error metrics can result in inaccurate models.

A hybrid modeling approach called Adaptive and Interactive Modeling System (AIMS) [LU90] is a unique metamodeling tool that integrates the inductive learning algorithms and statistics techniques with recursive decomposition and multiple objective optimization to find the best representation of the relationship (function) between the design variables and performance parameters. It has been successfully applied to build metamodels for vehicle bumper systems with success [LU98], and several large-scale, highly nonlinear engineering systems.

**Surrogate model-based design theories and methodologies**

Surrogate model-based design approaches use high-level, abstract model of an engineering system instead of the costly mechanistic model to conduct design. Figure 1 shows the generic process of surrogate model-based design procedure.

![Figure 1. Surrogate Model-based Engineering Design Process](Image)

The process starts with data collection, then metamodeling technique is used to build the surrogate models between a set of design parameters and performance parameters, and then these models are used to achieve the final design solution. The goal is to obtain robust design results in a more efficient manner than those approaches based on mechanistic models. The most popular surrogate model-based approaches are Response Surface Methodology (RSM) and associated design methodologies.
RSM is widely used for developing, improving and optimizing product and process. RSM comprises a group of statistical and mathematical techniques for empirical model building and model exploitation [BOX87]. For a given design space, different design parameter value combination (data) is selected through physical design of experiments (DoE) technique and least squares (LS) regression analysis is used to fit these data with a polynomial function, which is called response surface model (refer to 1.1). Then the response model is used as a prediction tool in the optimization to explore the design space seeking optimum design parameter settings [MONT75] [MYERS89] [MYERS95].

Since randomness assumption for least squares regression analysis is invalid for design processes that data is collected by deterministic computer experimental design, RSM is mostly used in physical experimental data collection-based design scenarios. Even for such situations, if the response models are not selected adequately, the following optimization may end up blind and endless search loops.

Some of the design methodologies try to combine RSM with Genichi Taguchi’s Robust Design Principles in order to improve design quality [ROSS88] [SIMPS97]. Unfortunately, they still can’t overcome the disadvantages that come with RSM itself mentioned above. Researchers in the “reverse engineering” use mathematical inversion to obtain design solutions based on certain assumptions [YAMA90]. Usually, these kinds of methods depend greatly on specific domain knowledge and can’t be applied to different conditions. New methods need to be developed in order to get reliable design results based on the surrogate models at the early stage of parametric engineering design.

BACKWARD MAPPING METHODOLOGY FOR DESIGN SYNTHESIS

Overview

At early stage of parametric engineering design, the goal is to find the feasible candidate(s) that satisfies the desirable performance requirement(s) in an initial design space. At this stage, however, the design information is still sketchy and design targets are constantly changing, and therefore a single point design solution that meets the performance requirements has little relevance. The common engineering practice is to specify targeted performance ranges and try to find set(s) of robust design alternatives that meets the specified performance requirements. This is particularly important for the large-scale engineering systems with highly nonlinear relationship between its design space and performance space.

Within BMDS framework, the performance requirements are a combination of designer-specified performance range and design confidence level. In order to identify the effective design regions, the overall design space is decomposed into feature subregions with much more comprehensible representations of relationship between design space and performance space. Then they are evaluated according to certain robustness criterion and the most robust design region(s) is selected as the result. In the following sections, the theoretical background and overall framework of BMDS are described.

THEORETICAL BACKGROUND

Basic assumptions and formulations

Let the complex, highly nonlinear relationship between the design space and performance space be expressed by an unknown function:

\[ y = f(x), \quad x = [x_1, x_2, \ldots, x_n]^T \in D \]  \hspace{1cm} (3)

In which, \( y \) is a set of \( m \) performance parameters, \( x \) is a set of \( n \) design parameters, \( f \) represents the relationship between them and \( D \) is a predefined design space, which consists of value ranges for the above design parameters. For each single performance parameter \( (y_j, j = 1, \ldots, m) \), designers need to find out the design solutions that can satisfy the required performance with desirable performance range and design confidence level. Thus, design synthesis problem is defined as:

**Definition 1:** For a specified performance value \( y_0 \in \mathbb{R}^m \), find out the most robust design region(s) \( \{x\} \subset D \) within which any design solution \( x \) will yield a performance value that lies within a certain specified range \( \bar{y}_0 \subset \mathbb{R}^m \) of \( y_0 \) with the probability of this outcome being at least equal to a specified design confidence level \( \delta_0 \in [0,1] \).

The relationship in equation (3) could be very complicated and the \( m \) performance parameters are often coupled. There are usually no explicit representation can be found to model \( f(x) \). In the BMDS framework, surrogate models are used to represent each individual relationship between \( y_j \) and \( x \):

\[ y_j' = f_j'(x), \quad x = [x_1, x_2, \ldots, x_n]^T \in D, \quad j = 1, \ldots, m \] \hspace{1cm} (4)

Two basic assumptions are used in BMDS:

**Assumption 1:** The design space \( D \) in equation (4) can be decomposed into \( k_j \) sub-regions and within each sub-region a linear function can be used to approximate \( f_j'(x) \).

Thus, \( y_j' \) can be represented using a piece-wise linear function:

\[ y_j' = a_j^T x + e = \sum_{l=1}^{n} a_{jl} x_j + e, \quad x \in D_l \subset D, \quad j = 1, \ldots, m; \]

\[ l = 1, \ldots, k_j \] \hspace{1cm} (5)

where, \( e \) is the error between function \( f_j'(x) \) and \( a_j^T x \), and \( D_l \) is a certain subregion.

**Assumption 2:** The error \( e \) is assumed to be identically distributed. The piece-wise linear function \( (a_j^T x + e) \) is assumed to be a spatial
stochastic function or a random function. Within each sub-region, the error distribution is assumed to be stationary.

A mathematical representation of Assumption 2 is:

$$y'_i \sim N(E(y'_i), V(e))$$

where $E(y'_i) = \mu_i \xi$ and $V(e) = \sigma^2$. This assumption holds true only when no perceptible trend or pattern is present and may be verified using normality plot.

Therefore, the performance requirements in the Definition 1 can be reduced to the following equation:

$$\{ x \mid P(y'_j \in [y_{j0} \pm r_{j0}, y_{j0} + r_{j0}] | x) \geq \delta_0 \}, \ j = 1, ..., m$$

For all the $x$ in the effective design regions, the possibility of all the corresponding $y'_j$ falling into the given performance range $[y_{j0} \pm r_{j0}, y_{j0} + r_{j0}]$ should be greater or equal to the specified design confidence level $\delta_0$.

**Overall framework**

As shown in Figure 2, the overall BMDS framework is realized by five major components: Domain Re-representation; Subregion Candidacy Screening; Effective Design Region Identification; Missing Region Recovery; Performance Vs. Design Parameters Robustness Evaluation of Design Region(s).

**Domain Re-representation** is the critical part of BMDS. Based on physical domain knowledge, an engineering system can be characterized by a set of most important design parameters and performance parameters with the associated value intervals. The complex relationship between these design parameters and performance parameters can be represented in a much more comprehensible form through metamodeling, which is called surrogate model [SIMPS97] (also refer to 1.1). AIMS is a hybrid metamodeling approach for forming empirical surrogates. Given a set of design parameters (inputs) and associated performance parameters (outputs), AIMS will build an empirical surrogate model to mimic the system behavior. For a complicated engineering problem with a large-sized design space, it is very difficult to find a good approximation over the entire design space. AIMS therefore uses a recursive decomposition method to split the design space into less complicated subregions and fits each region with an appropriate model. After AIMS operation, the complex, highly nonlinear relationship between design space and performance space is re-represented by a collection of more explicit and comprehensible form – piece-wise linear functions, which facilitates the backward mapping procedure. Detailed information and examples on how to use AIMS to build surrogate models can be found in [LU90] and [LU98].

**Subregion Candidacy Screening** is a process used to find all the qualified subregions that satisfy the targeted performance. The linear representation in each subregion is used to calculate the upper bound ($y_{j\text{max}}$) and lower bound ($y_{j\text{min}}$) of performance means of this subregion ($j = 1, ..., m; l = 1, ..., k_j$). For a certain given value of the performance parameter ($y_{j0}$), the subregion(s) with this value falling in between its performance range ($y_{j\text{min}} < y_{j0} < y_{j\text{max}}$) is selected as the Qualified subregion(s) ($D_q \subset D, q \leq 1, l = 1, ..., k_j$) for further investigation.

**Effective Design Region Identification** is conducted in each Qualified subregion. There are two very important parameters that decide the effective design regions. They are the lower bound of the effective performance mean: $y_{j\text{ql}}$, and the upper bound of the effective performance mean: $y_{j\text{qu}}$. These two parameters are determined by the given performance value: $y_{j0}$, the performance range: $r_{j0}$, the confidence level $\delta_0$, and the quality of the surrogate model in a qualified subregion. To find the two bounds, the following procedure is conducted.

First, check if there exists an effective design region based on performance distribution in each qualified subregion. If shaded area is larger than or equal to $\delta_0$, then there is a qualified effective design region (Figure 3).

$$\text{shaded area} \geq \delta_0$$

**Figure 3. EPR Qualification Check**

To find the $y_{j\text{ql}}$, move the performance mean distribution curve left such that the area under the curve and within the performance range is equal to the user-specified design confidence level: $\delta = \delta_0$ (Figure 4).
By the same token, $y_{j_0U}$ can be obtained by moving the curve right (Figure 5).

**Effective Design Region Identification** becomes straightforward after we convert the original model into piece-wise linear functions, selected all the qualified subregions and calculated $y_{jL}$ and $y_{jU}$. The direct inverse is used in each qualified subregion to find the effective design region(s):

$$y_{j_0} - r_{j_0} \leq \sum_{i=1}^{n} a_{ij} \leq y_{j_0} + r_{j_0}$$

$$x \in D_q \subset D$$

$$i = 1, \ldots, n; \ j = 1, \ldots, m; q \leq 1, l = 1, \ldots, k_j$$ \hspace{1cm} (8)

**Missing Region Recovery** is performed in case the effective performance mean bounds exceed the host subregion performance range. The issue does not arise if $\delta_0 > 0.5$. Otherwise, we explore all design subregions iteratively starting from the neighbors of all qualified subregions and evaluate $[y_{jL}, y_{jU}] \cap [y_{j_0} - r_{j_0}, y_{j_0} + r_{j_0}] (j = 1, \ldots, m; l \neq q$ and $l' \leq 1, l = 1, \ldots, k_j)$. If the result evaluates to null, we skip the subregion, otherwise it becomes a qualified subregion. Then effective design region identification procedure as mentioned above is conducted within each recovered qualified subregion.

**Performance Vs. Design Parameters Robustness Evaluation of Design Region(s)** is then conducted for each effective design region based on the robustness criterion. The robustness criterion represents the comprehensive sensitivity of performance space vs. design space. Currently, two kinds of criteria are used: 1) effective design region hypervolume; 2) maximum(minimum-dimension of design parameters) of the effective design region. The larger the criterion value, the less sensitive the performance parameter to the design parameters’ variations. The effective design region(s) with the largest robustness criterion value(s) is the most robust design region(s).

(i) Effective design region hypervolume criterion: If DR$_j$ is an effective design region in a given design space $D$ (DR$_j \subset D$), the design region volume for $x \in DR_j$ is defined as:

$$\int_{y_{jL}}^{y_{jU}} \cdots \int_{y_{jL}}^{y_{jU}} \prod_{i=1}^{n} dx_i$$ \hspace{1cm} (9)

Based on the tests so far, this criterion works effectively for any type of design region. One of the difficulties is, however, for high dimension design space the calculation of hypervolume is nontrivial.

(ii) Max(Min) criterion: It refers to the use of the minimum single design parameter value range within a effective design region as the robustness value for this region, and the design region with the maximum “minimum single design parameter value range” is the most robust design region.

Figure 6 shows an example for a 2D ($y = f(x_1, x_2)$) design space $D$. A, B, C are effective design regions identified by using BMDS. In A, there exist two design parameter value range, $r_{A1}$ and $r_{A2}$. Since $r_{A1} < r_{A2}$, the robustness value of A is $r_{A1}$. Similarly, the robustness values of B and C are $r_{B1}$, $r_{B2}$, $r_{C1}$, $r_{C2}$. C is selected as the most robust design region since it has the largest robustness criterion value $r_{C2}$. This criterion is expected to offer a simpler option for the design regions with relatively uneven single design parameter value range. The research work of its utilization conditions is currently in progress.

The above procedure can be used to realize “backward mapping” from performance space to design space for one performance parameter. In a design scenario with multiple performance parameters, the intersection of “backward
mapping” results for each single performance parameter is recommended as the final result.

As described above, BMDS offers a generic, efficient and robust framework to help designers make design decisions at early stage of parametric engineering design. And by combining a hybrid metamodeling approach with statistical formulation and techniques, BMDS has unique advantages in handling design synthesis problems with complex, highly nonlinear relationships.

**CASE STUDIES**

**A simple example**

Before applying BMDS to any real industrial design scenario, it is tested on a toy problem. A target engineering system is selected. And the relationship between its design space and performance space (10) is represented by an original function:

\[ y = x_1 + 2x_2 + (2-x_1)(2-x_2) \]  

where \(x_1\) and \(x_2\) are design parameters and \(y\) is a performance parameter. The design synthesis problem is described as:

\[ \text{Case I: } y_0 = 4.5, r_0 = 0.05, \delta_0 = 55\% \]
\[ \text{Case II: } y_0 = 4.5, r_0 = 0.10, \delta_0 = 85\% \]

BMDS is applied to each case to find the most robust design region(s) that satisfy the given performance requirements. Finally the results are validated.

**Results**

Table 1 shows the results from the first two major steps of BMDS. First of all, the hybrid metamodeling system AIMS is used to build more comprehensive representation for this nonlinear system. Uniform sampling is used to pick up 21 values for \(x_1\) and \(x_2\) within their range \([0.1, 2.1]\) respectively, and the cross-product between the selected data points forms the training data input set \((21 \times 21)\). This set is input into (10) to generate the training data output set. The input data with their associated outputs together form the training data set for AIMS. After AIMS operation, the nonlinear relationship between the design space and performance space (10) is represented by a collection of much simpler linear equations within 6 subregions. Among them, subregion \(D_0, D_1, D_2\) are identified as the qualified subregions after “Subregion Candidacy Screening” is conducted in each single subregion.

\[ \{ x \mid x \in [0.1, x_1 \leq 2.1 \land 1.1 \leq x_2 \leq 1.433333 \land 1.8860 \leq 0.3x_1 + 1.1x_2 \leq 1.9738 \} \]  

After BMDS operation, the final design results are:

1) In the first case, the most robust design region that satisfies the desirable performance between 4.45 and 4.55 with 55% confidence is:

<table>
<thead>
<tr>
<th>( y_1 ) and ( y_1 )</th>
<th>Effective Design Region(DR)</th>
<th>Design Region Hypervolume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 )</td>
<td>( y_1 )</td>
<td>Design Region Hypervolume</td>
</tr>
</tbody>
</table>

Table 2 shows the results of the other three major steps of BMDS. Within each qualified subregion, \( y_1 - y_0 \) is calculated; and it is combined with the associated subregion boundary and linear function to form the effective design region; then hypervolume for each effective design region is computed and used as the robustness index. The effective design region(s) with the largest robustness index value is the most robust design region(s). Two cases are shown in the following table.

Table 2. The Surrogate Model and Qualified subregions

Table 2. Design Synthesis Realization and Robustness Evaluation
2) In the second case, the most robust design region that satisfy the desirable performance between 4.40 and 4.60 with 85% confidence is:

\[
\{ x | 0.1 \leq x_1 \leq 2.1 \land 1.1 \leq x_2 \leq 1.433333 \land 1.8756 \leq 0.3x_1 + 1.1x_2 \leq 1.9879 \} \quad (12)
\]

**Validation of the results**

According to (7), the probability for the performance value of any design solutions within a certain effective design region falling between the required performance range should be equal to or larger than the specified design confidence. In order to verify that the above results satisfy this condition, a two-step validation procedure is conducted. First of all, testing examples (design solutions) are randomly selected in each subregion. Then the performances of these testing examples are input to matlab for performance distribution analysis.

1) Testing Examples Generation

In each case, more than 400 design points within each effective subregion are randomly selected as the testing examples.

2) Performance Value Distribution Analysis

The testing examples are feed into the original nonlinear relationship simulator (10) to get the corresponding performance values. Then these values are input to matlab for performance distribution analysis by using normplot.

**Table 3. Validation Results**

<table>
<thead>
<tr>
<th>Ply ∈ Y1 &amp; f1 ∈ [h]</th>
<th>x ∈ DR0</th>
<th>x ∈ DR1</th>
<th>x ∈ DR2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required performance</td>
<td>r01 = 0.05</td>
<td>0.544 &gt; 0.55</td>
<td>0.755 &gt; 0.55</td>
</tr>
<tr>
<td>range:</td>
<td>Design confidence</td>
<td>0.755 &gt; 0.55</td>
<td>0.9575 &gt; 0.85</td>
</tr>
<tr>
<td>Required performance</td>
<td>r01 = 0.10</td>
<td>0.8617 &gt; 0.85</td>
<td>0.975 &gt; 0.85</td>
</tr>
<tr>
<td>range:</td>
<td>Design confidence</td>
<td>0.8617 &gt; 0.85</td>
<td>0.975 &gt; 0.85</td>
</tr>
</tbody>
</table>

In Table 3, all the values at the left hand side of “>” (including 0.544) are the normplot analysis result, which are the probabilities of testing examples’ performances falling between required performance ranges for the associated effective design regions, and values at the right hand side are the given design confidence. It is necessary to point out that, since selection of testing examples affects the normplot analysis result, this result doesn’t necessarily represent the robustness level of a certain effective design region.

As shown in Table 3, all the effective design regions satisfy condition (7) except that the result of DR0 in the first case is a little off. This case study shows that BMDS provides reliable design solutions that satisfy the given performance requirements in this toy problem.

**Application to vehicle bumper system design**

In the summer of 1997, Ford Research Laboratory initiated a project to build surrogate models for vehicle bumper system design and analysis using AIMS. The goal was to find an alternative tool for the FEA simulation models traditionally used for bumper design and analysis. As a result, surrogate models are built between three key design variables and four critical performance parameters [LU98]. The validation with both FEA results and test results show that the quality of these surrogate models are very good. The same surrogate models are used to test the BMDS framework.

**Results**

Two of the surrogate models mentioned above are selected for the study: 1) maximum beam deflection (bxd) vs. vehicle weight (vW), beam cross-section thickness (tn) and foam density (fd); 2) maximum resultant rail load (rfl) vs. vehicle weight (vW), beam cross-section thickness (tn) and foam density (fd). The given initial design space D is: vW[3250, 3452] (lbs), tn[0.75, 2.00] (mm), fd[65.0, 90.0] (g/l) [LU98].

Within BMDS framework, the Bumper System Design Synthesis problem is described as: given the performance requirements 1) bxd = 35 mm with required performance range as [31, 39]mm and design confidence level as 80% and 2) rfl = 100,000N with required performance range as [96,000, 104,000]N and design confidence level as 80%, find the most robust design region(s) that satisfy these performance requirements. The results of each major step of Backward Mapping Methodology for Design Synthesis are shown in the Table 4, Table 5 and Table 6.

**Table 4. Surrogate Models for Maximum Beam Deflection (bxd) and Qualified Subregions**

<table>
<thead>
<tr>
<th>Bumper System Design and Analysis</th>
<th>Domain Re-representation</th>
<th>Subregion Candidate Screening</th>
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<tbody>
<tr>
<td>subregion boundary</td>
<td>linear equation</td>
<td>yes</td>
</tr>
<tr>
<td>surrogate models</td>
<td></td>
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<tr>
<td>beam deflection (bxd) vs. (vW, tn, fd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1) x20 &lt; vW &lt;= 3540</td>
<td>bxd &lt;= 0.007562vW</td>
<td>yes</td>
</tr>
<tr>
<td>x10 = 0.011x20+0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) x20 &gt; vW &gt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) x20 &lt; vW &lt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
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<td></td>
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<tr>
<td>4) x20 &gt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design and analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>beam deflection (bxd) vs. (vW, tn, fd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) x20 &lt; vW &lt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) x20 &gt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
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<tr>
<td>Performance requirements</td>
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<td></td>
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<tr>
<td>bxd &lt;= 35 mm</td>
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<td></td>
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<tr>
<td>tolerance: 4 mm</td>
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<td>confidence: 80%</td>
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<tr>
<td>x20 &lt; vW &lt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) x20 &gt;= 3540</td>
<td>bxd &lt;= 0.60352vW</td>
<td>no</td>
</tr>
<tr>
<td>x10 = 0.011vW+0.013</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 4, the surrogate model for maximum beam deflection (bxd) vs. vehicle weight, beam cross-section thickness and foam density is shown under the “Domain Representation”. The original design space is divided into seven subregions and in each of them, the system is represented by a linear function. For each subregion, the maximum and minimum beam deflection are calculated and
compared with the given deflection value (35 mm). The subregions 4 and 7 are regarded as the qualified subregions for the next step.

Table 5. Surrogate Models for Maximum Resultant Force on Rail (rf1) and Qualified Subregions

<table>
<thead>
<tr>
<th>Bumper System Design and Analysis</th>
<th>Domain Re-representation</th>
<th>Subregion CANDIDACY Screening</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subregion boundary</td>
<td>Linear equation</td>
</tr>
<tr>
<td>Surrogate model: resultant force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>on rail: rf1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs. (vw, tn, fd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training data: 63 data points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design space: 3250 - 3426 lbs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fn: 3250 &lt;= vw &lt;= 3452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design region robustness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Requirements: rf1 = 100,000 N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tolerance: 4,000 N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confidence: 80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Resultant Force on rail</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 6, surrogate representations in the qualified subregions are used to obtain the $y_L - y_U$ and Effective Design Regions for bxd and rf1 respectively. There are no missing Effective Design Regions in this case. Then Design Region Hypervolumes are calculated for each effective design region and the one with the highest hypervolume value is the most robust design region that satisfies the given performance requirements mentioned above for a certain performance.

According to design region robustness evaluation, the most robust design region for performance bxd requirements is DR$_b$ with the highest design region hypervolume 747.6541 and the most robust design region for performance rf1 requirements is DR$_f$ with the highest design region hypervolume 352.5872. Therefore, the final result is the intersection of DR$_b$ for bxd and DR$_f$ for rf1:

$$\{ X | X \in DR_b \cap DR_f \}$$

Validation of the results

In order to accurately validate the result, large data sets need to be generated by FEA model within the final result region for performance distribution analysis as in toy problem. Currently, such kinds of resources are unavailable. Thus the following approximate validation of the result based on available FEA data has been conducted:

1. Maximum Beam Deflection Vs. Thickness Validation

Given performance requirements (bxd = 35 mm, required performance range = 4 mm, design confidence = 80%), the associated design region (performance 31 mm – 39 mm) from FEA result is 1.195 < tn < 1.370.
2). Maximum Rail Deflection Vs. Thickness Validation

![Figure 8. Maximum Rail Deflection Vs. Thickness Validation](image)

Given performance requirements (rf1 = 100,000 N, required performance range = 4000 N, design confidence = 80%), the associated design region (performance 96,000 N ~ 104,000N) from FEA result is 1.160 < tn < 1.350.

In order to satisfy the both performance requirements, the final tn range from FEA data is: 1.195 < tn < 1.350, which is within the final tn range from BMDS result (1.166667 < tn < 1.375). Since the given design confidence is 80%, it is acceptable that there exists some regions that out of the FEA range. But any design solution within the BMDS result still stands 80% chance to satisfy the required performance range. This shows that Backward Mapping Methodology for Design Synthesis in this bumper system case can produce the effective design region that satisfies the given performance within a required performance range and with user-controllable possibility of success. Further validation is still in progress.

CONCLUSIONS AND FUTURE WORK

A Backward Mapping Methodology for Design Synthesis (BMDS) has been developed and tested for a real engineering application. BMDS is a design methodology for solving design synthesis problems encountered at the early stage of parametric design. It uses a hybrid metamodeling system AIMS to build surrogate models for a design problem and the statistic theory to derive the solution directly from the surrogate models based on the given performance requirements. It provides a systematic approach in handling inverse design problems and therefore avoids the iterations in the traditional design analysis approaches.

Our case studies demonstrate that the BMDS can be used as a quick design tool to assist engineers in finding design solutions for the given performance targets. Initial results show that BMDS’s assumptions are valid. AIMS can generate quality piece-wise linear functions to approximate design problems and the error distributions are generally following the “stationary normality distribution”.

Solving design synthesis problem is a challenging task. To do it effectively and efficiently is even harder. Combining surrogate models and statistic theory is the first attempt in achieving this goal. More research is needed to improve and enhance the BMDS theory/methodology. More design scenarios, especially real industry applications, need to be carried out to further validate the methodology.

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REFERENCES


